Bitcoin Spreads Like a Virus

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Abstract. We illustrate, by way of example, that Bitcoin's long-term price is non-random and can be modeled as a function of the logistic growth of number of users n over time. Using observed data for both Facebook and Bitcoin, we derive the relationships between price, number of users, and time, and show that the resulting market capitalizations likely follow a Gompertz sigmoid growth function. This function, historically used to describe the growth of biological organisms like bacteria, tumors, and viruses, likely has some application to network economics. We conclude that the long-term growth rate in users has considerable effect on the long-term price of bitcoin.

Keywords: bitcoin, cryptocurrency, valuation, growth model, network economics, network effect.

JEL Codes: C22, C49, C51, C53, C58, D85, E40, E42, E49, E51, G12, G17, L14

> Noise trading is trading on noise as if it were information. People who trade on noise are willing to trade even though from an objective point of view they would be better off not trading. Perhaps they think the noise they are trading on is information. Or perhaps they just like to trade…[T]hey don't know they are trading on noise. They think they are trading on information.
>
> —Fischer Black, *Noise*

If there were ever a noisy market, the young cryptocurrency environment fits the definition. Rarely do we witness such competing and sometimes concerted efforts to both establish and destroy a new asset class. The participants in this cacophony of change in the financial structure span the spectrum of naïve neophytes and phenomenal prodigies. At one end are young, arrogant day-trading chartists who seek to maximize their social media audience. At another are sophisticated, long-established financial institutions with millennia of combined experience and quantitative computing power beyond belief. In between are governments, consumers, opportunists, celebrities, crooks, and investors, each weighing in on cryptocurrency with their varying degrees of experience, means, and motive.

This paper offers a simple explanation of price formation in the burgeoning and oft misunderstood cryptocurrency ecosystem. Using bitcoin as an example, we provide convincing empirical evidence that price formation is not a semi-random result of emotional investing but instead is founded on economic principles of value that have only recently begun to be recognized: network economics.

For over a decade, bitcoin's price, as well as the viability and merit of the Bitcoin network itself, has been derided by classical economists as a fiction. Criticisms of bitcoin range from purely speculative investment to outright fraud. There is a common yet misinformed belief, even among ardent supporters and holders of bitcoin, that the true value of bitcoin is at best unknown and at worst not knowable or zero.

No modern currency, not even sovereign fiat currency, has intrinsic or fundamental value. Every currency in widespread use today is representative money. Commodity money has other characteristics that impart value, namely scarcity and durability, that paper money does not. However, lack of intrinsic value is not the same as having no value. The Federal Reserve Bank of St. Louis [Andolfatto, 2019] explains the lack of fundamental value is a perfectly normal and acceptable characteristic of both representative and commodity money:

> Gold, for example, trades above its value as measured by its industrial applications. The U.S. dollar trades above its fundamental value in discharging U.S. tax obligations. The premium some people are willing to pay for gold and the U.S. dollar reflects the value these objects possess as exchange media. The market value of these objects would decline, but not fall to zero, should this premium suddenly vanish.

An examination of bitcoin prices offers some interesting observations that directly counter the "value is a mystery" myth. The first is that, as proponents have long argued, the value of a currency is primarily dependent upon use and acceptance of that currency. Again, citing the Federal Reserve:

> What is the source of the fundamental value of Bitcoin? Think about it this way. At its core, Bitcoin is a database management system. Database management systems can have a fundamental value if they are tailored to meet the needs of a given constituency. Bitcoin offers people a money storage and transfer system with two key properties: (i) permissionless access and (ii) decentralized database management. The first property means that no one can prevent a user from sending any amount of Bitcoin from one account to another. The second property means that the protocol does not depend on the existence of a delegated authority to manage accounts and transfer funds. The

fundamental demand for Bitcoin derives from the fact that there are at least some people who value these features. This fundamental demand provides a non-zero lower bound on the price of Bitcoin.

This hypothesis has been tested and is also apparent from a cursory examination of the relationship between bitcoin's price and activity associated with the Bitcoin payment network. Specifically, price changes tend to be highly correlated with changes in number of wallets, active addresses, unique addresses, and transaction activity. Using daily data since January 2012 – February 2019, these correlations are summarized in Table 1.

Table 1

Correlations of Lognormal Returns in the Bitcoin Network

60-day periods from January 2012-February 2019

	price (USD)	*txn count*	*active addresses*	*unique addresses*	*wallets*
txn count	0.15				
active addresses	0.58	0.71			
unique addresses	0.44	0.64	0.73		
wallets	0.19	0.36	0.35	0.25	
transactions	0.20	0.85	0.63	0.84	0.28

Sources: coinmetrics.io and blockchain.info

By way of numerical example, we show that bitcoin's long-term price is a function consistent with the logistic growth of number of users *n* over time. From observed data, we derived the relationships between price, number of users (proxied as active addresses), and time, and show that the resulting market capitalization is consistent with a Gompertz sigmoid growth function.

The data were obtained entirely from coinmetrics.io and blockchain.info, and consist of daily prices and active accounts from July 2010 through February 2019.

THE NETWORK EFFECT IN BITCOIN

Metcalfe's law is based on the mathematical tautology describing connectivity among *n* users. Hence, network value *P* is a function of number of users *n*. As more people join a network, they add to the value of the network nonlinearly; i.e., the value of the network is proportional to the square of the number of users. The underlying mathematics for Metcalfe's law is based on pair-wise connections (e.g., telephony). If there are four people with telephones in a network, there could be a total of 3 + 2 + 1 = 6 connections. This law, like most other laws, assumes equality among the members' network connections. The law is commonly expressed in shorthand as n^2, which is the approximate value of *P* when *n* is large.

However, Metcalfe [2013] cautions that growth in *P* is subject to logistic decay, a function he labeled *affinity* (*A*). Alternatively, we could say that *n* grows nonlinearly consistent with a sigmoid function.

Compared to most economic theories, Metcalfe's law is relatively untested. Van Hove [2014] argued it was correct, while Zhang [2015], Peterson [2018], and Pele [2019] have all empirically confirmed Metcalfe's law. Peterson and Pele both asserted the law is only useful over long measurement periods, an unsurprising qualification in light of Black [1986] quoted at the beginning of this article.

The most commonly cited dispute of Metcalfe's law is Briscoe [2006], who argue Metcalfe's law as too optimistic and offer a subdued $n \log(n)$ model as an alternative. Their approach attempted to incorporate the concept of diminishing marginal return central to neo-classical economics. However, Briscoe [2006] does not derive the model and offers no empirical testing or results. Metcalfe [2013] counters that the diminishing incremental value is already captured in his *A* coefficient. Zhang [2015], Van Hove [2016] and Peterson [2018] concluded Metcalfe's law was a more parsimonious and quantitatively satisfying model.

Metcalfe's law determines the value of a network for a given *n*. It says nothing about the growth rate of *n*. We consider three possibilities:

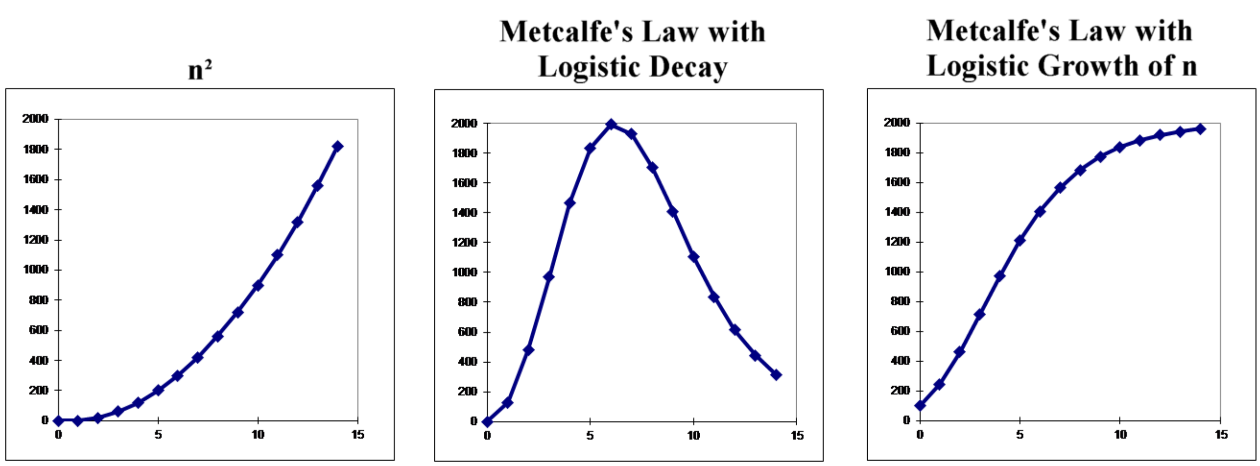
- If *n* grows linearly, then $P \approx n^2$. Metcalfe has said this assumption is unrealistic as *P* would grow without bound. However, in periods of early adoption, this model is probably a close approximation of value.
- *P* is bounded by logistic decay.
- *n* is subject to logistic growth. This approach is the subject of this paper.

It is likely than *n* grows at a small rate, then a large rate, then a small rate, indicating stages of adoption as indicated in telephony by Chaddha [1971], Bewley, [1988], and Islam [2002]. Zhang [2015] used a similar function to model Facebook and Tencent market capitalizations. This bell-shaped pattern of growth rates, on a cumulative basis, gives rise to a sigmoid function:

$$P_t \approx n_t^2 \approx \alpha e^{\beta e^{\gamma n}} \tag{1}$$

To illustrate the differences, generic plots of three variations of Metcalfe's law are show in Figure 1. The first is the simple approximation of Metcalfe's law. The second is Metcalfe's revision to his law by incorporation of a logistic decay factor *A*, and the third is a Gompertz function.

Figure 1



A NUMERICAL EXAMPLE USING BITCOIN AND FACEBOOK

We start by observing two relationships integral to the pricing of bitcoin.

First, bitcoin's lognormal price P with respect to time t is a near-perfect horizontal parabolic arc. A horizontal parabola is

$$x = a(y - k)^2 + h \tag{2}$$

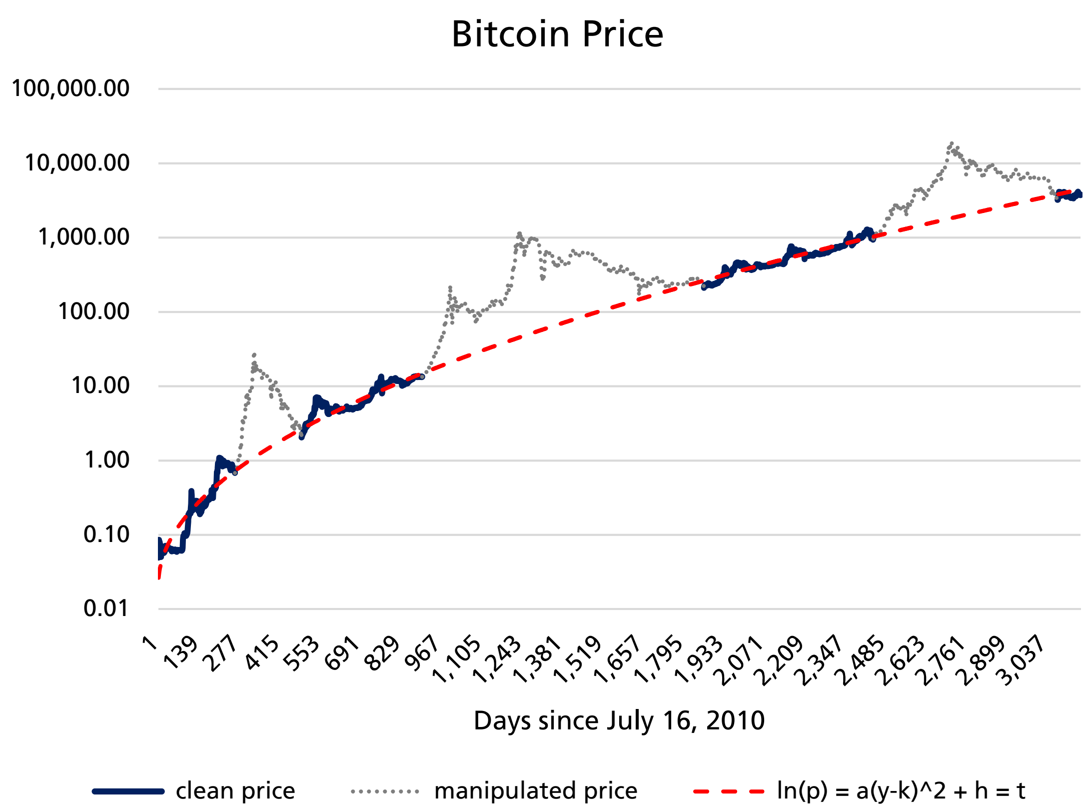
where (h, k) is the vertex. Plotting $\ln(P_t)$ and setting $a = 1$, $h = 0$, and $k = 0$ we have

$$\ln(P) = 1\left(\sqrt{t} - 0\right)^2 + 0 \tag{3}$$

$$\ln(P) = t \tag{4}$$

which we have graphed in Figure 2.

Figure 2



The elegance of this relationship between price and time is by no means a coincidence. It is rare that one has an opportunity to view the gradual adoption of a currency (or other asset) over time. This is in part because many companies are private during their development stages. But using well-known Facebook (FB) we can see it follows a similar pattern (Figure 3).

Figure 3

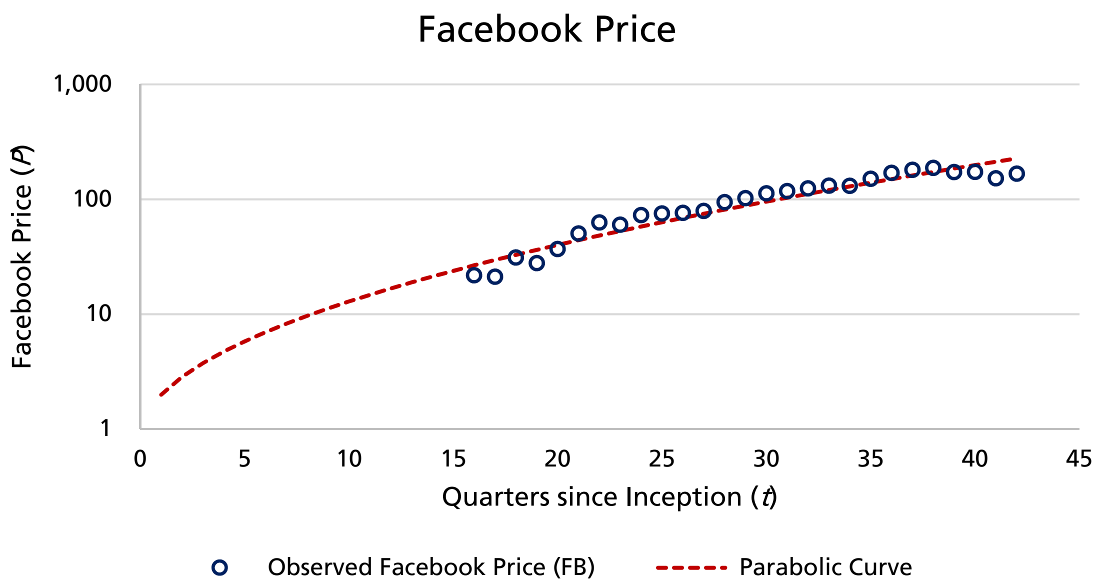
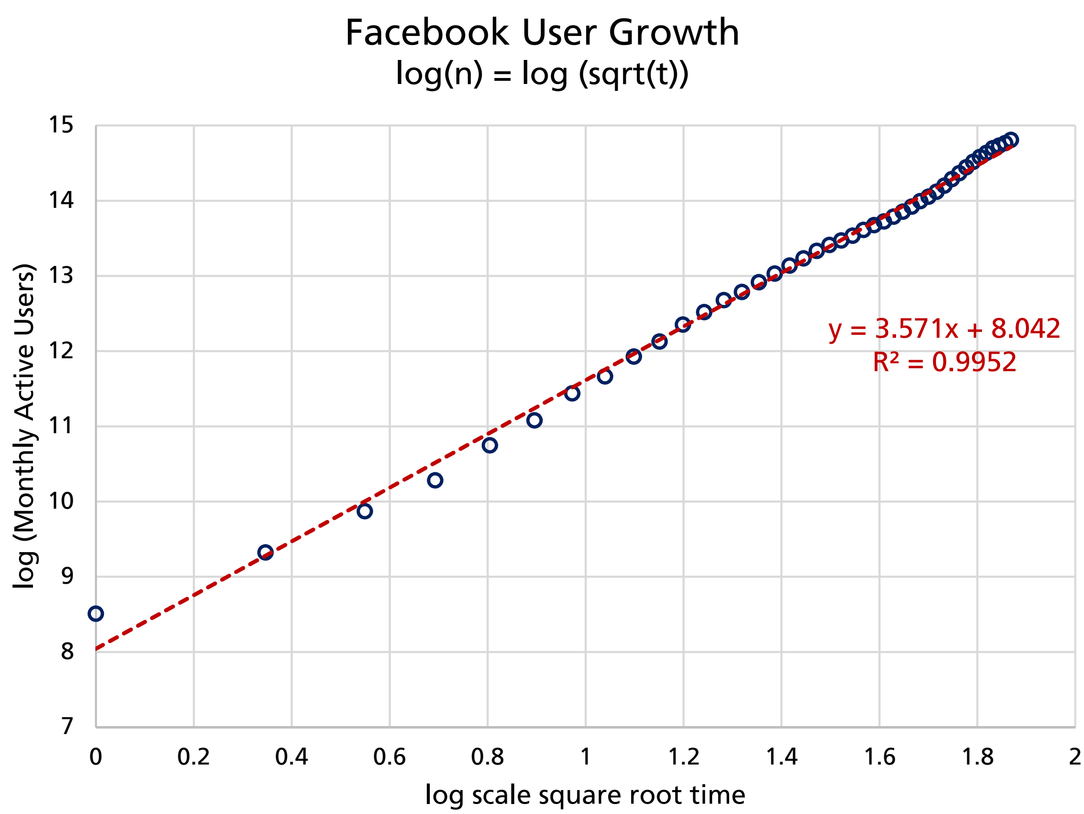


Figure 4



Source: https://www.statista.com/statistics/264810/number-of-monthly-active-facebook-users-worldwide/

The relationship in Figure 3 is a consequence of Metcalfe's law. In Figure 4, we plot the Metcalfe value P of Facebook Monthly Active Users (n) where

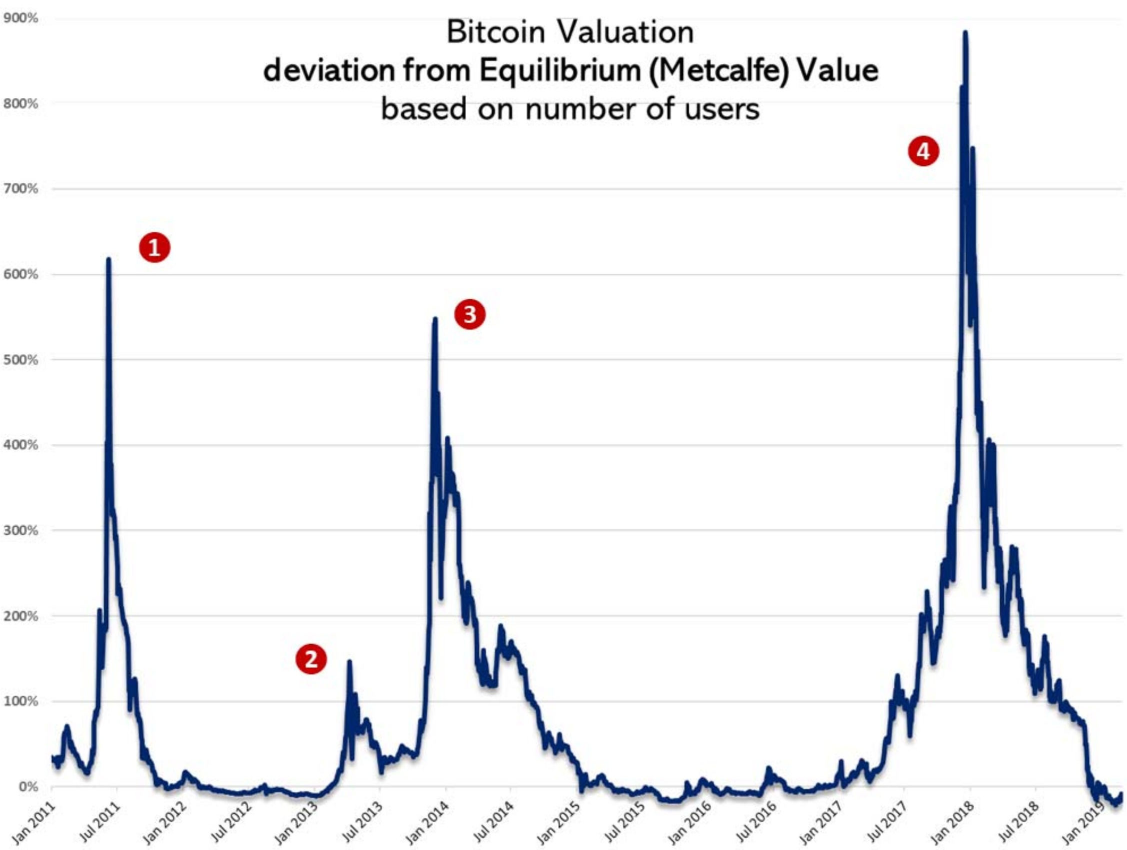
$$P = \frac{n(n-1)}{2} \tag{5}$$

As we will see later, the relationships that describe Bitcoin's value follow the exact same mathematical growth principles as those of Facebook. Facebook Figure 3 corresponds to Bitcoin Figure 1. Facebook Figure 4 corresponds to Bitcoin Figure 7.

Facebook pricing prior to the IPO is unavailable. The value can only be estimated with Metcalfe's law, and Figure 4 lays the groundwork for that analysis. We do not analyze Facebook's value in this paper. We only use it to show the persistence of the network effect outside of cryptocurrency.

Referring back to Figure 2, there are three notable exceptions where bitcoin's price deviated from the parabolic trend. These are periods of documented price manipulation and eventual resolution to equilibrium. These periods of price manipulation have been well-researched and described in Gandal [2017], Griffin [2018], and Peterson [2018], as well as the financial press [Leising, 2018 and Robinson, 2018]. Using the method described in Peterson, we plot the deviation from equilibrium value to better illustrate the four periods of price manipulation, shown in Figure 5. Accordingly, we exclude these periods (identified by local price minima preceding and subsequent to drawdown) from our analysis.

Figure 5

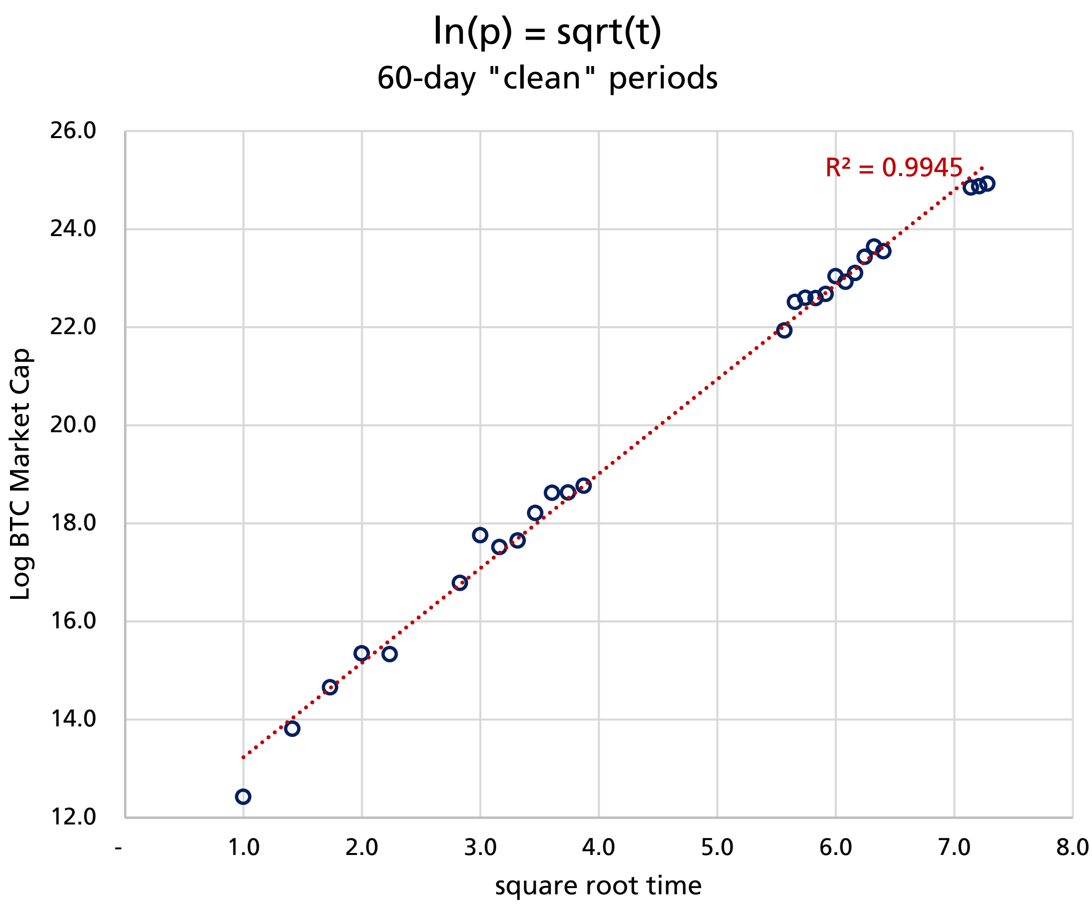


Using periods where Bitcoin's price was in equilibrium, we can easily see

$$\ln(P) = a\sqrt{t} + b \tag{6}$$

which we plot in Figure 6.

Figure 6



Growth in Bitcoin Users

From Figure 7, we observe that $n = f(t)$:

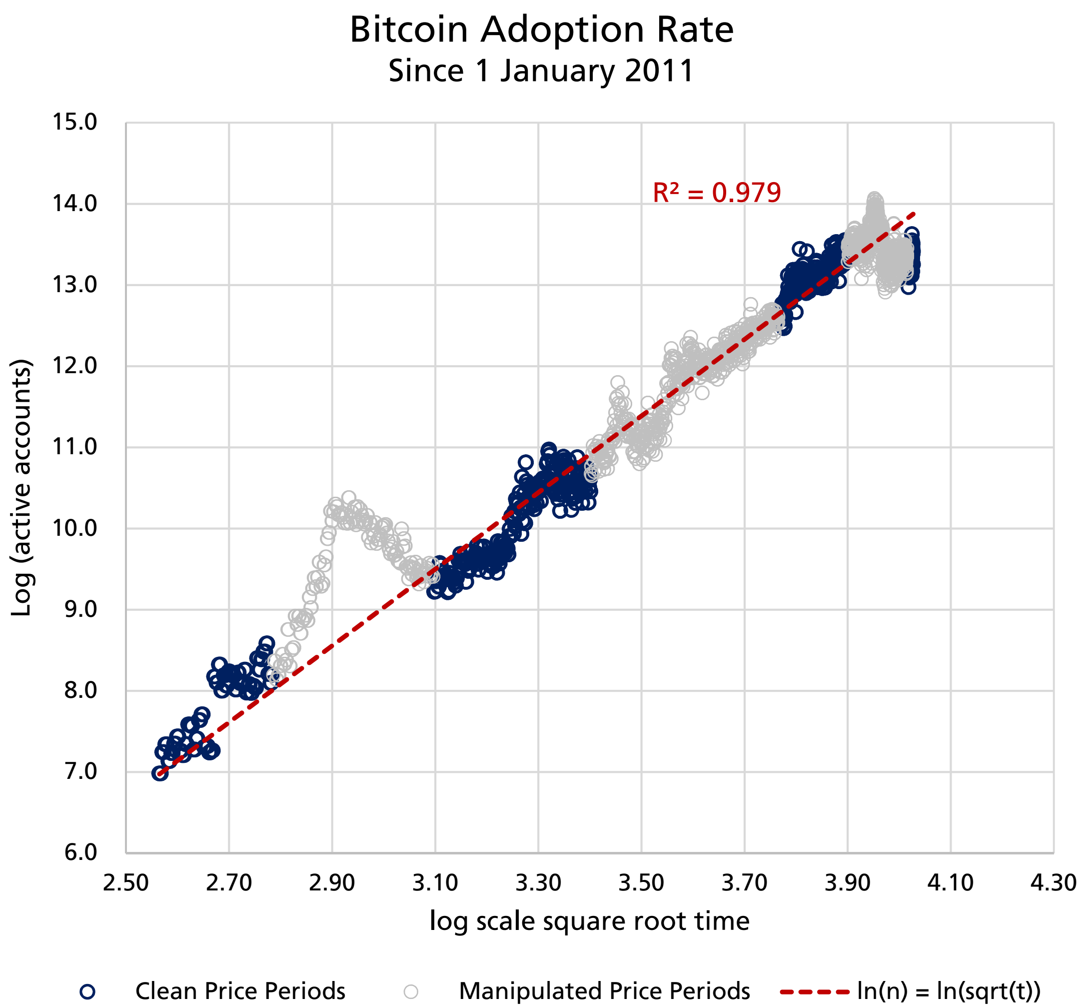
$$\ln\left(\sqrt{t}\right) = c\ln(n) + d \tag{7}$$

This relationship can also be expressed as

$$\sqrt{t} = e^{c\ln(n)+d} \tag{8}$$

Substituting Equation 8 into Equation 6, we have

Figure 7



$$\ln(P) = ae^{c\ln(n)+d} + b \tag{9}$$

which is in the form

$$P = \alpha e^{\beta e^{\gamma n}} \tag{1}$$

where

$$\gamma \approx \%\Delta n \tag{10}$$

As previously mentioned, Equation 1 is a Gompertz function, a logistic function that for decades has been used to model viral infection [Scott, 1959], bacterial growth [Murphy, 1973], tumor growth [Brunton, 1980], and mobile phone proliferation [Islam, 2002]. The most notable application of a Gompertz function to bitcoin pricing to date is Peterson [2018] who used it to model Metcalfe's affinity coefficient.

Relationship to Metcalfe's Law

Using coefficient notation from Equations 5 and 6, and using possible paired connections notated as n^2, the relationship between network value and network size can be expressed as

$$\ln[\ln(P_t) - b] = c'\ln(n_t^2) + d' + \varepsilon \tag{11}$$

We plot this relationship in Figure 8.

Analysis of Bitcoin Price and User Data

Using a linear first-differences approach (60-day intervals) to control for autocorrelation in the time series, we obtain estimates for *c'* and *d'* in Equation 11 and present the results in Table 2 and Figure 9. This regression is constrained such that the intercept *d'* = 0, a condition we found likely to be true when running an unconstrained regression.

Figure 8

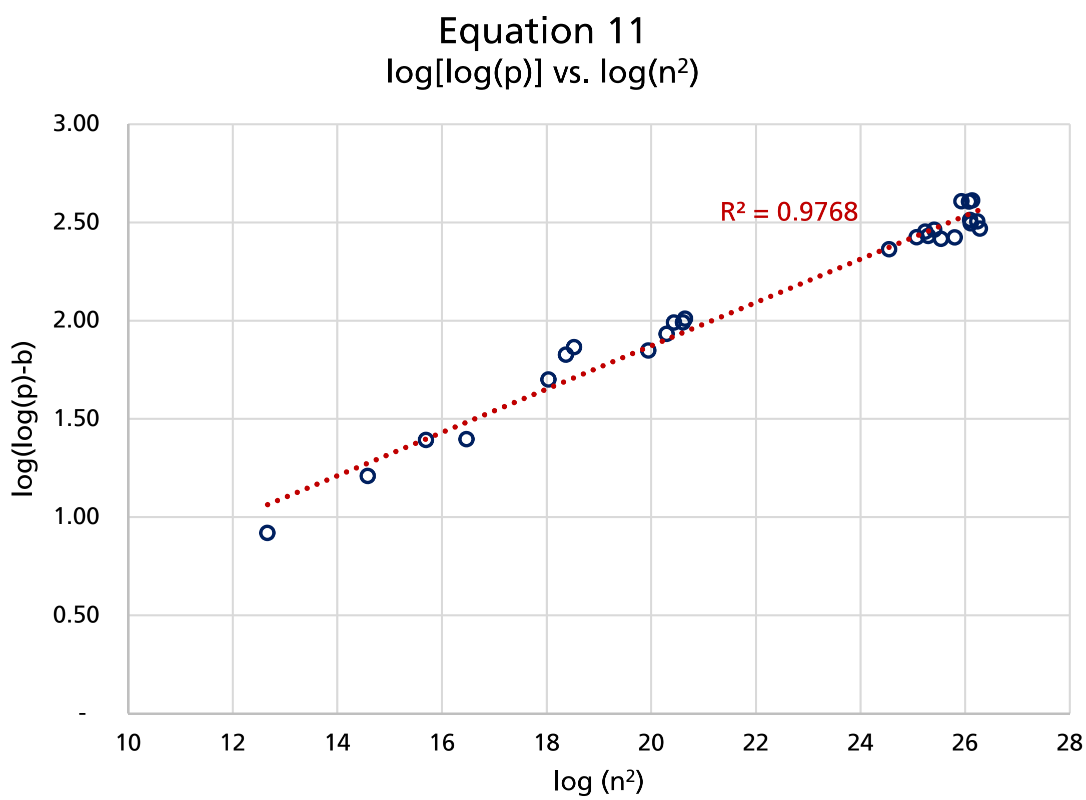
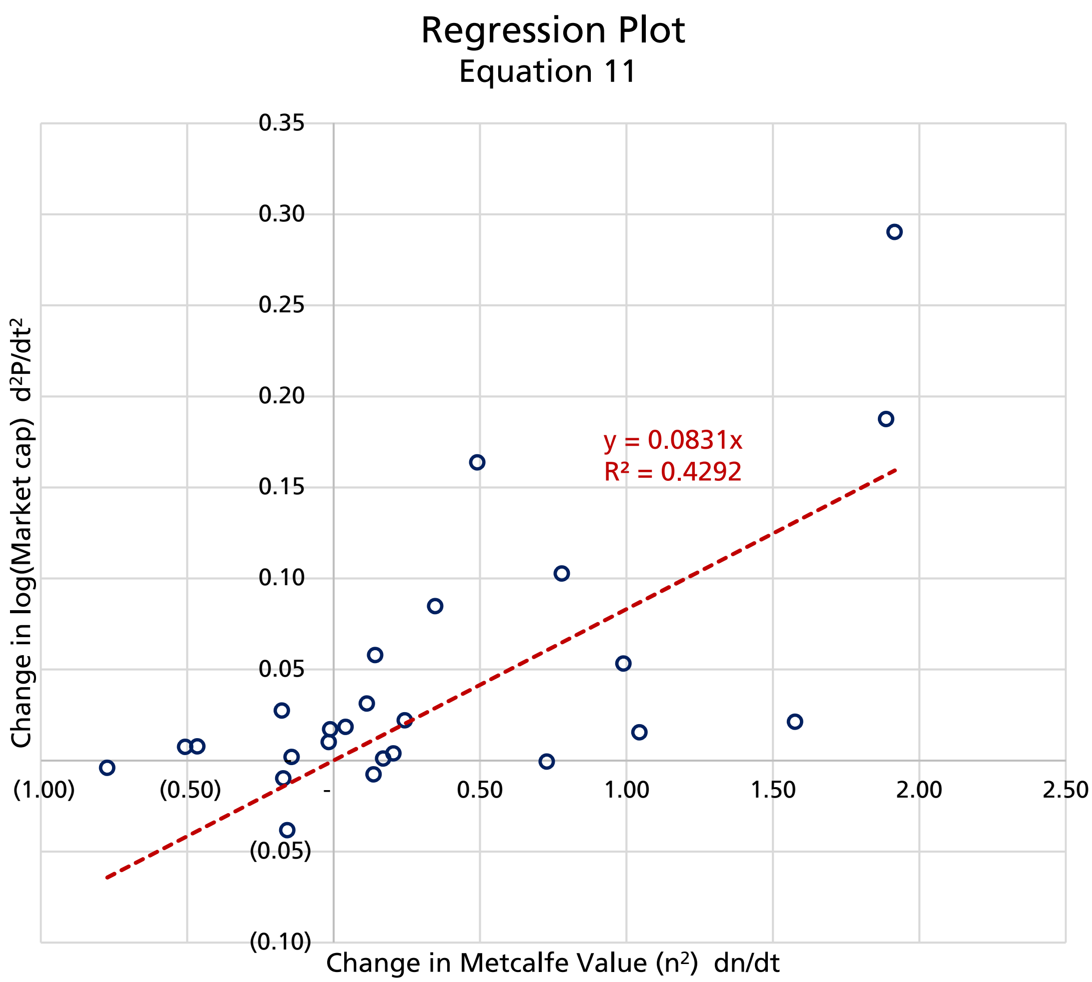


Table 2

Regression Statistics					
R Square	0.58				
Standard Error	0.06				
Observations	25.00				
Durbin-Watson	1.80				
	Coefficients	*S.E.*	*t Stat*	*P-value*	
Intercept (d')	0.0000				
Slope (c')	0.08	0.01	5.72	0.00	95% conf.

Figure 9



CONCLUSION

A key application of our findings is the ability to evaluate data and marketplace news with the intent to separate meaningful information from misleading noise. Noise is a dominant driver of price and volatility in the short-term. It is not noteworthy that markets disseminate and consume noise about bitcoin or any other cryptocurrency. Forecasts of sky-high prices and doomsday crashes are common with bitcoin. In many cases, cryptocurrency information (positive and negative) is crafted in a convincing way by experienced and knowledgeable sources and presented by reputable media. In isolation, these predictions are indistinguishable from information, even though they are probably noise.

Though value is not observable, even an imperfect assessment of value serves to keep markets efficient. Over time price tends toward value. The model we have presented serves as a backdrop against which potential information can be evaluated. It does not predict that bitcoin's price will soar or crash. Rather, it suggests that the *probability* of those extreme those events is very small because ultimately number of users drives price.

To date, the typical approach to cryptocurrency valuation has been via Metcalfe's law. Commonly expressed in shorthand as n^2, it is the approximate value of P when n is large. We show that price is a function of n users, as Metcalfe's law states. Our research differs from past models in that we derive that n may grow at a non-constant rate over time, as a Gompertz function would indicate. This function, usually used to describe the growth of biological organisms like bacteria, tumors, and viruses, likely has some application to network economics, including cryptocurrency valuation. Lastly, we confirm past research that the long-term growth rate in users has considerable effect on the long-term price of bitcoin.

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